HOW TO LOOK AT THE CUSANUS' GEOMETRICAL FIGURES?

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Introduction

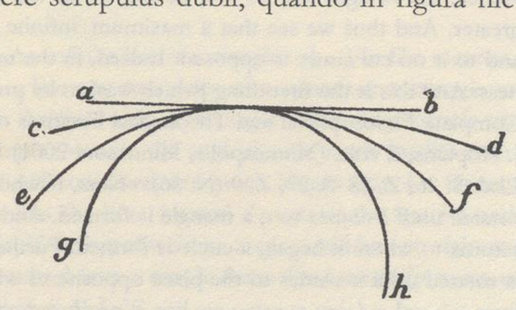
It's difficult to read Nicolas of Cusa's mathematical writings for several reasons: first, the proofs are not very clear, but above all, the figures are not easy to understand. Cusanus doesn't know the technique of drawing in perspective and he adopts some conventions we don't understand today. For example, he represents a cone by a right-angled triangle, with its apex at the bottom and its basis on a circle [document 2]. Sometimes, the copyist turns over the figure, from the top to the bottom or from the right to the left. But the most important difficulty is that these figures, which are necessarily set on a paper sheet, are conceived as movable ones. The figures of the *De docta ignorantia* are well known [I, 13, 14 and 15; s. document 1].

Document 1

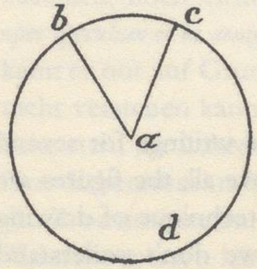
De docta ignorantia I, 13

De passionibus lineae maximae et infinitae

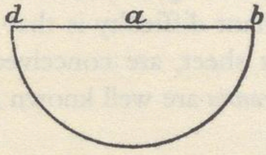
[. . .] Nec hic potest remanere scrupulus dubii, quando in figura hic lateraliter videtur, quomodo arcus *c d* maioris circuli plus recedit a curvitate quam arcus *e f* minoris circuli, et ille plus a curvitate recedit quam arcus *g h* adhuc minoris circuli; quare linea recta *a b* erit arcus maximi circuli, qui maior esse non potest. Et ita videtur, quomodo maxima et infinita linea neces-



sario est rectissima, cui curvitas non opponitur, – immo curvitas in ipsa maxima linea est rectitudo; et hoc est primum probandum [. . .][1]



Secundo, si linea *a b*, remanente puncto *a* immobili, circumduceretur, quousque *b* veniret in *c*, ortus est triangulus; si perficitur circumductio, quousque *b* redeat ad initium ubi incepit, fit circulus. Si iterum, *a* remanente immobili, *b* circumducitur, quousque perveniat ad locum oppositum ubi incepit, qui sit *d*, est ex linea *a b* et *a d* effecta una continua linea et semicirculus descriptus. Et si, remanente *b d* diametro immobili, circumducatur semicirculus, exoritur sphaera;[2]



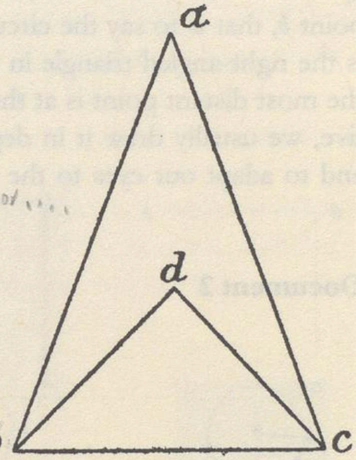
[1] *De docta ign.* I, 13: h I, S. 26, Z. 11–20 (N. 35): »The characteristics of a maximum, infinite line: [. . .] Not even a scruple of doubt about this can remain when we see in the figure here at the side that arc *c d* of the larger circle is less curved then arc *e f* of the smaller circle, and that arc *e f* is less curved then arc *g h* of the still smaller circle. Hence, the straight line *a b* will be the arc of the maximum circle, which cannot be greater. And thus we see that a maximum, infinite line is, necessarily, the straightest; and to it no curvature is opposed. Indeed, in the maximum line curvature is straightness. And this is the first thing [which was] to be proved [. . .]« *On Learned Ignorance*, in: Complete Philosophical and Theological Treatises of Nicholas of Cusa, translation by J. Hopkins, 2 vols. (Minneapolis, Minnesota 2001) I, 21.

[2] Ebd. S. 26, Z. 28–S. 27, Z. 9 (N. 36): »Next, if while point *a* remains fixed, line *a b* is rotated until *b* comes to *c*, a triangle is formed. And if the rotation is continued until *b* returns to where it began, a circle is formed. Furthermore, if, while *a* remains fixed, *b* is rotated until it comes to the place opposite to where it began, viz., to *d*, then from lines *a b* and *a d* one continuous line is produced and a semicircle is described. And if while the diameter *b d* remains fixed the semicircle is rotated, a sphere is formed [. . .]« *On Learned Ignorance*, translation by J. Hopkins (as quoted in n. 1) I, 21–22.

De docta ignorantia I, 14

Quod infinita linea sit triangulus

[. . .] videre poteris triangulum lineam esse quanti sint simul iuncta tanto tertio longiora, quanto angulus, quem faciunt, est duobus rectis minor, ut angulus *b a c* quia duobus rectis multo minor… longiores *b c*. Igitur quanto angulus ille maior fuerit, ut *b d c*, tanto minus vincunt lineae *b d* et *d c* lineam *b c*, et superficies minor. Quare si per positionem angulus valeret duos rectos, resolveretur in lineam simplicem totus triangulus.[3]



These figures are not only simple lines always put together in the same manner; proofs and comments joined to the geometrical figures in his mathematical writings show us that Cusanus saw them in movement. What are these motions? Could we recreate them? Is it possible, today, to recreate exactly what Cusanus saw on his figures?

We shall rapidly examine some examples in order to deduce some theoretical conclusions. I have chosen characteristic examples in the second book of the *De mathematicis complementis* (1454) [documents 2, 3, 4, 5, 6].[4]

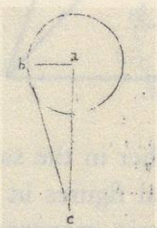
[3] Ebd. S. 28, Z. 23–31 (N. 39) – An infinite line is a triangle : »[. . .] In like manner, you can see that a triangle is a line. For any two sides of a quantitative triangle are, if conjoined, as much longer than the third side as the angle which they form is smaller than two right angles. For example, because the angle *b a c* is much smaller than two right angles, the line *b a* and *a c*, if conjoined, are much longer than *b c*. Hence, the larger the angle, e. g., *b d c*, the less the lines *b d* and *d c* exceed the line *b c*, and the smaller is the surface. Therefore, if, by hypothesis, an angle could be two right angles, the whole triangle would be resolved into a simple line.« *On Learned Ignorance*, translation by J. Hopkins (as quoted in n. 1) I, 23.

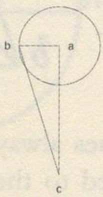
[4] For these documents, I use the edition of Basel of the *Opera* (Nicolaus de Cusa, *Opera* [Basel 1897] = b) and the Monacensis 14213 codex (fol. 105^{r}–108^{v}).

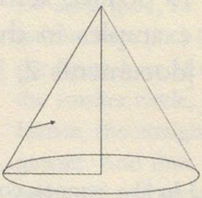
I. Some examples of the generation of a figure

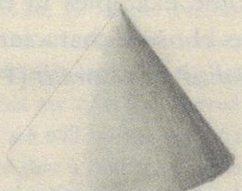
The generation of a cone [s. document 2] may disconcert us because the apex is at the bottom. Yet, the matter for Cusanus is the course of the point *b*, that is to say the circumference of a circle. The object in motion is the right-angled triangle in the foreground. Its way is vertically drawn, the most distant point is at the top while, since the invention of perspective, we usually draw it in depth. We have to adjust our way of looking and to adapt our eyes to the ancient conventions.

Document 2









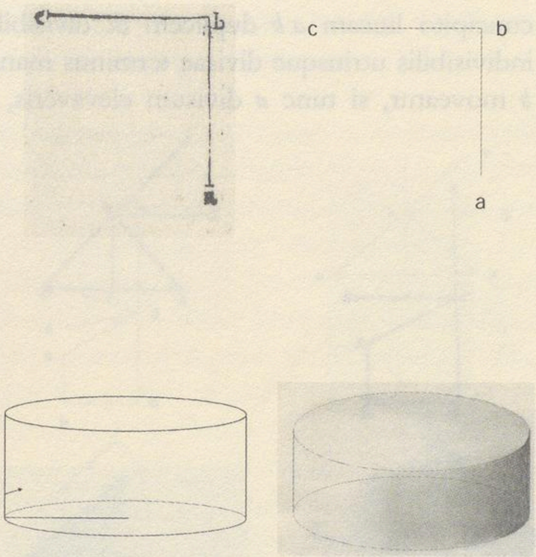
[. . .] superficies circuli, habens semidiametrum ut quatuor: ad superficiem illius quae habet semidiametrum ut 2, quadrupla est. Quae conicarum superficierum ad invicem, & ad suas bases, ex hoc habetur. Nam cum semidiameter basis, & latus trianguli quod conicam describit superficiem, moveantur uno terminali eorum puncto fixo, & super eadem basis circumferentia: illa erit superficierum habitudo quae linearum, ex quarum motu ipsae superficies constituuntur, uti est semidiameter basis & latus illud trianguli, ex quo conica describitur superficies ut *a b* & *b c*.[5]

[5] *De mathematicis complementis*: b 1034. »[. . .] the surface of circle the radius of which is like four is four surfaces of circle the radius of which is like two. Hence, we have the ratio of the conical surfaces to their basis and vice versa. Indeed, since the radius of the basis and the side of the triangle which describes the conical surface are moving, with one of their ends fixed on the circumference of basis, so, the ratio of the surfaces

The generation of a section of a column [s. document 3] simply consists of two lines on a right angle. *a* is the centre of the basis circle. *ab* is the radius. *c b* is the height of the cylinder. Today, we draw *a b* horizontally and *b c* vertically. But, since *b c* moves away from our eyes, *b c* is raised vertically.

Document 3

linea quae basim efficit, movetur uno puncto eius terminali stante, alio circumferentiam describente: & illam et columnarem superficiem constituit, per motum aequalem utriusque terminalis puncti, super eadem circumferentia basis. Ut ex *a b c* angulo recto super *a* circumvoluto, describitur basis per *a b*, & per *b c* duplex superficies Cylindrica: quia *b c* aequalis *a b*, aequaliter in *b c* punctis terminalibus movetur.[6]



The generation of a section of a column topped by a cone [s. document 4] is even more difficult to read. Point *a* turns around *b*. Segment *b a* is drawn on several positions while it describes a circular plane

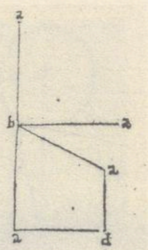
will be the ratio of the lines with the movement of which the surfaces are built. For example, the radius of basis and side of the triangle which describes the conical surface are *a b* and *b c*.« (My translation).

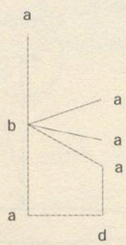
[6] Ebd.: b 1035. »the line which generates the basis is moved when one of its ends is fixed and when the other end describes a circumference; and this line builds a surface of a column by the equal movement of each of its ends on the circumference of the basis. If angle *a b c* turns round *a*, the basis is described by *a b*, and the double cylindrical surface is described by *b c*, because *b c* is equal to *a b*, and *b c* moves equally on its ends *b* and *c*.« (My translation).

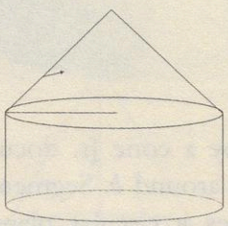
around *b*. When the distance is the most important from the eye, it forms a single vertical line *a b a*. However, in the same figure, Cusanus represents a lower position of *a*, so that *b a* draws a cone. Therefore, we have to look at the same line *b a* altogether in two different ways, as a flat circle and as a cone.

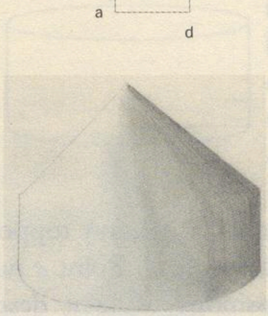
Document 4

concipito lineam *a b* duplicem & divisibilem usque ad *b* punctu: qui indivisibilis utriusque divisae terminus maneat. Esto igitur quod *a* stante *b* moveatur, si tunc *a* divisum elevaveris, ut circa *b* fiat angulus: tunc secundum circumferentiam quam *a* mobile describet, ad circumferentiam quam *b* describit, scire poteris proportionem superficierum. Puta esto quod *a* mobile elevetur ut constitat talem angulum, quod linea quae de *a* cadit usque ad punctum, qui ita distet ab horizonte sicut *a* fixum & sic *a d* fit medietas *a b*: tunc *b a* mobile describet superficiem conicam, quae erit maior plana circulari, quam *a b* describit pro medietate, et ita proportionabiliter in omnibus. Quare placet, quod quando *a* mobile elevatur, ut eius motus fit duplex ad motum *a b* (scilicet quando erit ex ipsis linea una) tunc *a b* mobile describet superficiem triplam, et planam ad superficiem quam *a b* describit.[7]







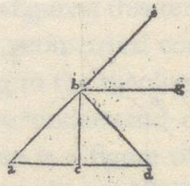


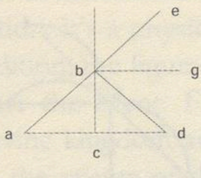
[7] Ebd.: b 1036–1037. »I see line *a b* as a double line, divisible at the point *b* which is indivisible. *a* moves when *b* is fixed. If you raise *a* so that you get an angle around *b*, then you could know the ratio of the surfaces according to proportion between the circumference described by *a* and the circumference described by *b*. For example, *a* is raised to build an angle so that the line dropped from *a* to the point which is horizon-

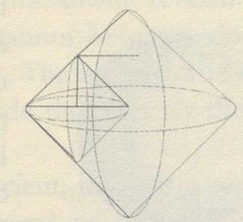
The generation of a rhombus [s. document 5], that is to say two cones opposite to each other by their basis, accumulates all the previous difficulties. The small cone is described with *a b* while point *a* runs over the small circle the radius of which is *b c*; so *b* is the most distant point from our eye. But, at the same time, triangle *a b d* pivots around *a d* and forms a rhombus; point *b* comes near our eye. The things are more and more complicated with the next hypothesis (*b e* and *b g*).

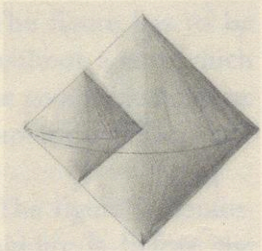
Document 5

a b c triangulus fit, & *a b* latus describens conicam, & *c b* semidiameter basis: trahe lineam *a c* in continuum, & de *b* duc lineam ut facias aequalem triangulum, qui fit *b d c*. Manifestum est, si *a d* fixa manente, circumvoluitur triangulus *a b d*, rombum oriri ex duobus aequalibus conicis. Trahe igitur *a b* in continuum, & fit *b e* ut *a b*: clarum est si circumvoluitur ut prius, lineam *b e* efficere superficiem triplam ad superficiem *a b*, & co-









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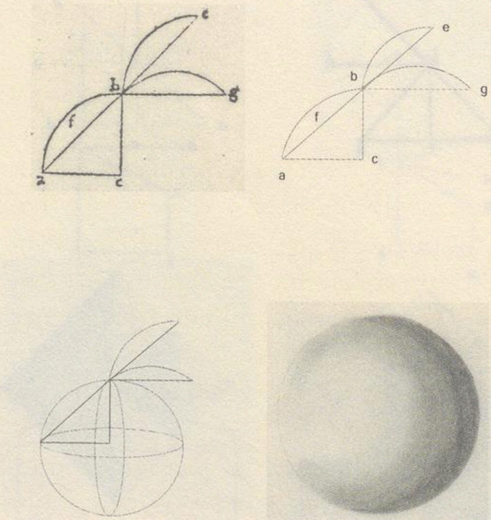
tally distant from *a* makes *a d* the half of *a b*. Then *b a* is moved and describes a conical surface which will be half longer than the surface described by *a b*, and so proportionately in all cases. That's why, when *a* is raised so that its movement makes two movements of *a b* (and *a b a* will be a single line), then *a b* describes a plane surface three times as large as the surface described by *a b*.« (My translation).

nicam superficiem *a e* quadruplam esse ad eam quae & *a b*. Unde si *b d* elevaveris in medium inter *b d* & *b e*, & fit *b g*: efficiet superficiem duplam, sicut *b d* aequalem, & *b e* triplam.[8]

Finally, the generation of a sphere [s. document 6] is described as a variant of the generation of a rhombus. Instead of straight lines, arcs are turned. Arc *a f b* will generate a semi-sphere when the point has run the whole circumference whose radius is *c b*. If *b* turns in front of our eyes, we shall have the whole sphere. According to Cusanus, the other arcs give larger spheres with known proportions.

Document 6



si feceris latus coni chordam arcus, describendo arcum super ipsum, ut super *a b* latus *a f b* arcum, & super *b e* eandem arcum: erit superficies ex curva *a f b* tertia superficiei, quae ex curva *b e*. Et ita si volueris duplam, facito ut in conicis dictum est.[9]

[8] Ebd.: b 1037. »*a b c* is a triangle, side *a b* describes a conic, and *c b* is the radius of the basis. You draw continuously *a c* and you draw from *b* a line so that you build an equal
d triangle, viz. *b d c*. It's obvious, if *a b* remains fixed when triangle *a b d* turns around, that the result is a rhombus with two equal cones. So, you draw continuously *a b* that makes *b e* like *a b*. It's obvious, if you make the rotation as before, that *b e* generates a surface three times as large as the surface generated with *a b*, and that *a e* generates a conical surface four times as large as the surface generated with *a b*. Hence, if you raise *b d* in the middle between *b d* and *b e*, viz. *b g*, the result is a double surface; with *b d* an equal surface; with *b e* a triple surface.« (My translation).

[9] Ebd.: b 1037–1038. »If you turn the side of the cone into a chord of an arc on which

Nicolas of Cusa is interested, as you know, in the way to shift from a figure to another one with determined proportions. He wants to discover the law which allows to generate figures. What does »to generate a figure« mean? It's impossible to observe the generation of a figure in nature. This operation is possible only in the mind, with a particular insight. To understand how we have to look at a figure, I suggest to compare the observation of the geometrical figures with the observation of the »omnivoyant« that Nicolas of Cusa explains in *De Icona* (or *De visione dei*).

II. The figure in *De Icona* and the geometrical figures

In both cases, we use the word »figure«; this term refers at the same time to the idea of a face and to the geometrical configuration; it's a physical form made in a work. According to the theological tradition, the figure is a prefiguration whose sense we understand only after the event. For example, the temple of Jerusalem is a figure of the divine kingdom and Adam is a figure of Christ. According to Saint Paul, the events of the Old Testament happened to be exemplary figures for Christians.[10] The figure is not a simple symbol. It's a picture whose sense the Christians have to discover thanks to the divine revelation. The figure has to be decoded; it's a mystery; it cannot be given alone, without a text which accompanies and explains it. The same is true of the geometrical figure: it's not only a drawing beside the text; it's the geometrical object considered in the text.

The figure isn't self-sufficient; it needs a sense. The figure is contingent and variable, as the name of things. The quiddity is before the figure. Nicolas of Cusa says that the figure or the mystery we have to decode is the means to approach the quiddity, on condition that we take it in our mind.[11] In Nicolas of Cusa's words, the figure is first a form, an outline. In the *De docta ignorantia*, he discusses the figure of the earth:

the arc is itself described, like on side *a b* the arc *a f*, and like on side *b e* the same arc, so the surface described by the curve *a f b* will be the third of the surface described by curve *b e*. In the like manner, if you would have a double surface, I should describe it as I have said about conics.« (My translation).

10 1 *Corinthians* 10, 11.

11 *De mente* 6: h ^{2}V, N. 92.

»sphaera est ultima perfectio figurarum, qua maior non est«.[12] The sphere is the best representation of the divine maximum. However, the earth isn't perfectly spherical because it cannot be equivalent to God himself; it only tends to a sphere.

Notice that in Nicolas of Cusa's time, a geometrical figure is not a simple set of lines, but is the object contained in these lines. For example, the circle is not a centre with a circumference, but is the surface area contained in the circumference (which we name, today, a disc).

On a painting as on a geometrical figure there is a representation with two sides: first, the object fixed on the paper, motionless, which we carefully examine, the dimensions of which are determined with the artist's pencil; secondly, the object which comes to life in a motion; on the painting of the *De Icona*, life is obtained with the motion of the spectators; in the mathematical writings, motion is inside the mind. Thanks to the motion of the onlookers, the painted eyes of the »omnivoyant« come to life; thanks to geometrician's mind, the geometrical figure comes to life, too, because it's moved. The painting of the *De Icona* is unique and fixed on a wall, but it will come to life thanks to the multiplicity of monks who will observe it when they move. The same is true for each geometrical figure: it's a unique and particular figure, but it will be observed by a multiplicity of readers, and will be connected in the mind with a multiplicity of cases.

If the figure is watched without instructions, it's banal and doesn't reveal any particular signification, exactly as Rogier Van der Weyden's painting would not reveal any particular signification if Cusanus didn't give instructions on the manner to observe it. What do we seek when looking at this painting? It's a strange experiment about reciprocity: the portrait fascinates us because it gives the impression of looking at us, too. I can't obtain this experiment with a mirror because, when I look at myself in a mirror, my eyes aren't fixed on a particular point of the plane; so, I really see only my eyes, not a gaze with subjective thought. In the character on the painting, indeed, I don't recognize myself, but I recognize myself as somebody looking; so, in this way, I exist. To see and to be seen are the

[12] *De docta ignorantia* I, 23: h I, S. 46, Z. 24 (N. 71); »[The] sphere is the ultimate perfection of figures and is so that than which there is no more perfect.« *On Learned Ignorance*, translation by Jasper Hopkins (as quoted in n. 1) I, 38.

same, and so, I begin to see the divine presence. The painted face of the »omnivoyant« is a figure for representing infinity; thanks to his skill, the painter has managed to give some universality to the eyes of his figure:

> Video in hac picta facie figuram infinitatis. Nam visus est interminatus ad obiectum vel locum et ita infinitus. Non enim plus est conversus ad unum quam alium, qui intuetur eam. Et quamvis visus eius sit in se infinitus, videtur tamen per quemlibet respicientem terminari.[13]

In spite of the limits of this form, the painter has managed to suggest the infinite and creative power of God. More exactly, through this painted figure shown by the artist, God shows himself.

III. The mirror of the mind

What do we seek when we look at geometrical figures? It's not a divine revelation but something else. The geometrical figure doesn't look at me. So, I have to observe it inside my mind. We know that Cusanus frequently compares the mind with a mirror. We have to add immediately that it's a living mirror; this mirror doesn't only passively record the pictures, but it also makes the notions exist.

The mirror of the mind is a reflective and positive power which reflects itself. What is this mirror like? It's a plane mirror,[14] like the paper sheet on which Cusanus draws his figures. We have to conceive the mind like a tablet put up vertically, on which concepts are reflected, exactly like a painting in which what is pictured is constantly changing. However, since it shows motions in space, this mirror has a depth.

How does it work? This inner mirror doesn't content itself with passive impressions or with received forms, because the mind is able to conceive the forms in themselves (for example, the circle in itself): these forms

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[13] *De vis. Dei* 15: h VI, N. 61, Z. 5–9: »In this [icon's] painted face I see an image of infinity. For the gaze is not confined to an object or a place, and so it is infinite. For it is turned as much toward one beholder of the face as toward another. And although in itself the gaze of this face is infinite, nevertheless it seems to be limited by any given onlooker.« *The Vision of God*, translation by J. Hopkins (as quoted in n. 1) II, 709.

[14] Proclus, in his *Comment about the first book of the Euclid's Elements*, compares also the mind to a plane mirror (Proclus, *Commentaire du premier livre des Eléments d'Euclide*, trad. Paul Ver Eecke [Paris 1940]).

don't have physical existence.[15] Like malleable wax,[16] the mind conforms to experienced things and creates their forms. Knowledge is an assimilation process through which the mind becomes similar to its objects:

> anima [. . .] quapropter ut multitudinem discernat unitati seu complicationi numeri se assimilat et ex se notionalem multitudinis numerum explicat. Sic se puncto assimilat qui complicat magnitudinem, ut de se notionales lineas superficies et corpora explicet. Et ex complicatione illorum [vel illarum] scilicet unitatis et puncti mathematicales explicat figuras, circulares et polygonias, quae sine multitudine et magnitudine simul explicari nequeunt.[17]

The mind is a reflective view, is a mirror which looks at itself. Nicolas of Cusa compares the mind with the point of a diamond where the forms of all things reflect themselves.[18] When the mind looks at itself, it gives to itself the concepts of the things. The mind changes from a passive mirror into an active mirror and creates the forms. Cusanus uses the metaphor of the mirror so that we can find the mind on two sides: beside the senses, the mind receives impressions; beside the ideas, the mind creates forms.

What are the products of the mind ?

> [. . .] mens nostra [. . .] facit assimilationes formarum, non ut sunt immersae materiae, sed ut sunt in se et per se, et immutabiles concipit rerum quidditates utens se ipsa pro instrumento sive spiritu aliquo organico, sicut dum concipi circulum esse figuram, a cuius centro omnes lineae ad circumferentiam ductae sunt aequales, quo modo essendi circulus extra mentem in materia esse nequit [. . .] Unde circulus in mente est exemplar et mensura veritatis circuli in pavimento.[19]

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15 *De mente* 7: h ^{2}V, N. 102–103.

16 Proclus (as quoted in n. 14) compares also the mind to wax.

17 *De ludo* II: h IX, N. 92, Z. 10 and 13–19. »The soul [. . .] assimilates itself to oneness, i. e., to the enfolding of number. And from out of itself the soul unfolds a multitude's conceptual number. Likewise, the soul assimilates itself to a point which enfolds magnitude in order to unfold from itself conceptual lines, conceptual surfaces, and conceptual three-dimensional figures. And from the unfolding of those things, viz., of oneness and of point, the soul unfolds geometrical figures (both circular and polygonal) which cannot be unfolded without both multitude and magnitude.« *The Bowling-Game*, translation by J. Hopkins (as quoted in n. 1) II, 1231.

18 *De mente* 5: h ^{2}V, N. 85–86.

19 Ebd. 7: h ^{2}V, N. 103, Z. 1–11: »Our mind [. . .] makes assimilations of forms not as they are embedded in matter but as they are in and of themselves. And it conceives the immutable quiddities of things, using itself as its own instrument apart from any instrumental [corporeal] spirit, as, for example, when it conceives a circle to be a figure from whose centre all lines that are extended to the circumference are equal; in this

The soul looks inside itself, produces both the mathematical concepts and the sciences which examine them.[20] When geometry mentions the circle, it's neither a perceptible thing nor a unique form in the understanding. There's only one circle in the understanding, indeed, but geometry examines a multiplicity of circles. Geometry brings together all the circles to one general matter, but the circle in itself cannot be divided in the understanding. Geometry considers a general matter and, through the imaginary circles, it examines another circle, the circle which is in the understanding. About the Proclusian geometry, between physical and intelligible things, Stanislas Breton writes it's an immaterial practice: *the geometrical being is the sign of an action.*[21] This being is both existing and concluded; it's not a drawing because the Euclidian line is devoid of thickness; we have to set a line apart from the mark of a pencil; it's the ideal sign of a mental operation. Rule and compass are instruments from mental product. Likewise, according to Cusanus, the geometrical figure is a sign of a mental action.

What effect will the mathematical work have on the mind itself? Thanks to the thinking on the figures, the mind will discover itself. In this progress, the mind acquires knowledge both about things and itself:

> quapropter mens ipsa, quae figuras in se intuetur, cum eas a sensibili alteritate liberas conspiciat, invenit se ipsam liberam a sensibili alteritate. Est igitur mens a sensibili materia libera et habet se ad figuras mathematicas quasi forma.«[22]

As the »omnivoyant« painting reveals to me that I am God's child, so the geometrical figure reveals to the mind its purity and its intelligence.

Nicolas of Cusa uses other metaphors to describe the mind. For example, he uses an incorrect etymology and considers the mind as an

way of existing, no circle can exist extra-mentally, in matter [. . .] Hence, the circle in the mind is the exemplar, and measure-of-truth, of a circle in a patterned floor.« *The Layman on Mind*, translation by J. Hopkins (as quoted in n. 1) I, 558.

20 Ebd. 9: h ^{2}V, N. 116–117, and *De ludo* II: h IX, N. 92.

21 St. Breton, *Philosophie et mathématique chez Proclus* (Paris 1969) 62 and 67 (The translation is mine).

22 *De theol. compl.*: h X/2a, N. 2, Z. 17–21: »since the mind, which views figures in themselves, beholds them as free of perceptible otherness, it discovers that it, itself, is free of perceptible otherness. Therefore, the mind is free of perceptible material and it stands in relation to mathematical figures as being their form.« *Complementary Theological Considerations*, translation by J. Hopkins (as quoted in n. 1) II, 748.

instrument for measuring. »Mentem quidem a mensurando dici conicio«;[23] the mind (*mens*) gets its name from »to measure«; it's a »measurement«; this figurative expression is deliberately equivocal, as it is both the operation of measuring and the rule to estimate things. »Rationalium vero praecisio intellectus est, qui est vera mensura.«[24] The mind owns the standard with which it measures the truth.[25]

Nicolas of Cusa also compares the mind to a living compass which would measure all things. Mathematics is the characteristic work of the mind. If we represent the mind as a wax tablet, we have to add it's a tablet written on from inside; it is a tablet which writes itself on itself because it contains living ideas, that is to say ideas which move themselves.

IV. The nature of the mathematical objects

Now, we are able to deduce some conclusions about the nature of the mathematical objects. They are objects put in a very hierarchical genealogy. These objects generate each other so that, when you know the proportions that enable you to measure them, you are able to go from one to the other. Geometry generates its notions in a definite hierarchy that we can find in Euclid's definitions: oneness, limit, point, line, surface, angle, circle, and so on. The indivisible point generates the divisible line; the widthless line generates the surface, and so on. Notice that the main problem – the solution of which Cusanus searches through his mathematical works – is the problem of the quadrature of the circle. When he begins to examine this problem, he wants to demonstrate the power of his principle of the coincidence of opposites. He hopes to find the proportion which allows to go from straight lines to curved ones (and vice versa). That's why he makes »geometrical transmutations«, that

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23 *De mente* 1: h ^{2}V, N. 57, Z. 5–6: »I surmise that mind [mens] takes its name from measuring [mensurare].« *The Layman on Mind*, translation by J. Hopkins (as quoted in n. 1) I, 535.

24 *De coni.* I, 10: h III, N. 52, Z. 11–12: »Now, intellect, which is a true measure, is the preciseness of things rational.« *On Surmises*, translation by J. Hopkins (as quoted in n. 1) I, 189.

25 Proclus (as quoted in n. 14), Prologue.

is to say he tries to pass from one form to the other, for example from a rectangle to a triangle, from the quarter of a circle to a triangle, from a column to a parallelepiped, and so on.

Mathematical objects are pure and finite objects. In mathematics, the mind aims at finite things. It subtracts infinity from pluralities and magnitudes and put them in finite proportions. This »restriction« of things in the finite domain gives its importance to proportion. It's essential to contain things in limits since objects enfold each other: each being which generates other beings causes all these beings. The cause not only determines, but also enfolds the thing that is caused. In the hierarchy of beings, there's this principle: the generating principle is richer than the generated principle. In the Cusanus' theory, this principle gives the strict hierarchy between straight line and curved lines.

Conclusion

The practice of geometry doesn't enable us to reach divinity while the observation of the »omnivoyant« gives an analogy to the sighted of God; but it allows glimpsing at infinity, and, above all, the immense variety of the living forms. The geometrician's mind experiences, on a finite scale, God's creation in the infinite scale. Today, we receive Cusanus' writings with his invitation to imitate the monks of Tegernsee. Van der Weyden's painting was burnt in a fire; the geometrical figures remain but seem inert, like dead. We have to awake them and to revive them in our mind.